SELF-REGULATED LEARNING IN THE MATHEMATICS CLASS

Charles Darr and Jonathan Fisher

Paper presented at NZARE Conference, Turning the Kaleidoscope,

Wellington, 24-26 November, 2004.

INTRODUCTION

Developing a sense of agency as a learner is at the heart of Self-Regulated Learning (SLR). If students in mathematics are going to become self-regulated learners they need to be confronted with opportunities that allow them to reveal their thinking and to observe and emulate the thinking of others. In what follows we begin by introducing SRL and exploring its connection with mathematics education.

In our small-scale classroom study, two elements of instruction in mathematics stood out as providing rich opportunities for students to begin practising self-regulatory behaviours. The first was the use of models to represent problem situations and the second, reflective journalling. We describe the exploratory study and in particular, examine how involving models and journalling as part of instruction, enabled students to observe and emulate self-regulating behaviours.

WHAT IS SELF-REGULATION

Self-Regulated Learning (SRL) refers to research and theory that has emerged since the mid-1980's concerned with how students, " … become masters of their own learning processes" (Zimmerman, 1998, p.1). A self-regulated learner is someone who is actively involved in maximising his or her opportunity and ability to learn. This involves not only exerting control over cognitive activity (metacogniton), but also developing metavolitional skills that enable the regulation of attitudes, environments and behaviours to promote positive learning outcomes.

According to Zimmerman (1998), SRL involves three major cyclical phases: forethought, performance control, and self-reflection. The first of these, forethought involves analysing tasks and setting appropriate goals. Performance control refers to monitoring and controlling the cognitive, behavioural, emotional and motivational acts that affect performance. The third phase, self-reflection is concerned with making judgements about what has been accomplished and altering behaviours and goal orientation accordingly.

The ability to self-regulate is highly correlated with success as a learner. The skills it involves are teachable and according to Zimmerman (cited in Pape and Smith, 2002, p.94), students with appropriate scaffolding, progress through stages of observation, emulation and self-control before finally arriving at what can be called self-regulation. Whether students develop or apply SRL skills however, is heavily influenced by their judgements of self-efficacy, their beliefs about the subject matter and their motivations.

SELF-REGULATION AND MATHEMATICS EDUCATION

Developing the ability to self-regulate is integral to the socio-cultural theories of learning that have influenced mathematics curricula over the last fifteen years or so. Traditional learning goals that focused on the mastery of facts and procedures have made way for objectives that emphasise understanding, flexible thinking, communication, and problem solving. Students are now expected to develop self-regulatory knowledge and skills that allow them to interact with mathematical ideas in an active and constructive way.

Problem solving is the area within mathematics education where the direct application of self-regulatory skills is most apparent. To actively make sense of problem situations, expert problem-solvers employ a fully self-regulated approach: analysing, planning, exploring and reflecting. In comparison, naive problem solvers are much more haphazard, spending a minimum of time planning or analysing, and using 'hit and miss' approaches (Schoenfeld, 1992). Research shows that students who lack self-regulation skills often rely on direct translation methods to solve problems. These involve recognising certain key words to transform text into equivalent mathematical sentences (Pape & Wang, 2003). Naïve problem solvers are also often plagued by the phenomenon of inert knowledge. This is knowledge that is available in student's minds, but which can not be accessed or applied when it is needed to solve new problems. Some researchers go as far as to argue that it is classroom practices themselves that are often responsible for students acquiring inert knowledge (Van Haneghan et al, cited in De Corte, 2000, p.709).

Developing the kinds of pedagogy that leads to SRL in mathematics learning and problem-solving has proved to be problematic. Despite the influence of socio-cultural theories on official curriculum documents, a transmission model of teaching and learning that emphasises teacher-regulation often predominates in classrooms. Pape et al (2003) argue moreover, that at times a lack of explicit teacher guidelines in socio-cultural pedagogies also hinders the development of SRL. They advocate programmes of instruction that combine socio-cultural ideas with the principles of SRL.

Several researchers have attempted to explore how classroom environments can support the development of self-regulation in mathematics (De Corte et al. , 2000; Pape et al., 2003; Schunk 1996, 1998). In a review of some of this research, De Corte et al (2000, p.196), list three components of instruction that appear to foster self-regulation: realistic and challenging tasks; variation in teaching methods including teacher modelling, guided practice, small group work and whole class instruction; and classrooms that foster positive dispositions towards learning mathematics.

The small-scale study reported in this paper was also designed to explore how a learning and teaching environment could support SRL, in this case, in the area of proportional reasoning. Our initial look at the literature had convinced us that encouraging students to report and explain their thinking would be a central feature of such an environment. We were also informed by some previous work we had done in another classroom exploring how instruction could support the development of proportional reasoning. In the next section we describe the methodology used to carry out the study.

THE SMALL SCALE STUDY

The study took the form of a teaching experiment and was conducted with a Year 7 class in a large, mid-decile intermediate school. Carried out at the beginning of the school year, students had come to the intermediate from various contributing schools and had a wide range of mathematics education experiences. The researchers, in partnership with the classroom teacher planned and taught twelve lessons over a four-week period based around the area of proportional reasoning.

SRL is highly relevant to proportional reasoning. Becoming a proportional reasoner involves learning to recognise and strategically co-ordinate the elements that make up proportional relationships. A skilled proportional reasoner is able to initiate a wide range of general problem-solving heuristics and monitor their progress on their way to a solution. Percentages was chosen as the area of proportional reasoning to focus on. In this, our approach was stimulated by the work of Moss and Case (Moss and Case, 1999). Joan Moss and Robbie Case developed an innovative rational number curriculum that began with instruction on percentages. Moss and

Cases' work was predicated on the idea that students' everyday knowledge of percentages and their intuitive ideas about proportions could be used to foster powerful understandings of the rational number system.

Taking this lead, we developed lessons that invited students to apply their intuitive knowledge and understandings about percentages and proportions to meaningful problems. Real and imaginable contexts were developed that we hoped would connect with students' experiences and motivate them to engage in problem-solving behaviours. Most critically, we hoped that classroom discourse (of both students and teachers) would model and support self-regulating behaviours.

Activities were designed at the whole class, group, pair and individual level and time was also provided for students to write journal entries reflecting on their learning. We also designed an interview protocol that was used with five students at the beginning and end of the study. Before and after the study students took a short test. Most of the questions were written to test elements of proportional reasoning. Some of the problem types used in the test were not covered in the lessons.

During classroom sessions, one of the researchers co-ordinated the lesson or lesson section, while the other videotaped the unfolding events in the classroom. The video camera was often carried around the classroom and used to record interactions with and between students. Data was also collected from several other sources including (1) artifacts from planning; (2) field notes; and (3) student journals and workbooks.

Our experiences in the classroom helped inform us about the way the principles of SRL can help focus teaching and learning. In the next section we begin by looking at the initial state of self-regulation within the classroom. Then, based on our analysis, we describe how thinking models embedded in rich mathematical tasks, together with reflection through journalling, opened up opportunities for students to examine their own and others' mathematical learning and problem solving.

THE INITIAL STATE OF SELF REGULATION

Both the preliminary interviews and pre-test suggested that most of the students showed little self-regulation as proportional reasoners. When a well-known strategy, such as successive halving, could not be applied to quickly answer a problem, there was little or no further investigation. In many cases, students simply reverted to using inappropriate arithmetical operations or provided a guess.

In the following interview transcript, Irene demonstrates this lack of regulation. Although, to start with, she shows that she can confidently manipulate halves and quarters, as soon as she moves out of this 'comfort zone' Irene resorts to guessing.

Interviewer: Let's go back to this one. How do you know that a quarter of one hundred is twenty five?

Irene: Because, going back to the question about how many twenty-fives in a hundred. That's four times twenty five is a hundred. So I just went back to the four.

Interviewer: And what about this one? What's a fifth of one hundred?

Irene: What did I say?

Interviewer: I think you said 5.

Irene: I just looked at one fifth and sort of guessed it was five.

There was plenty of evidence however, suggesting that Irene had access to knowledge that could have informed her attempt to find a fifth. Later in the interview, she explained that she could form a picture of one fifth of a hundred, even if she could not calculate it.

Interviewer: What do you think someone's asking you, when they're trying to make you get a fifth of something?

Irene: Well when someone asks me that. Go to a pie.

Interviewer: Oh. Show me.

Irene: [Draws a circle.] This is the pie. If someone asks for a fifth, I'd cut a fifth off.

... it's that [pointing to a fifth section].

... and so, I, that's one hundred [indicates the entire pie] and that's a fifth [indicates the fifth section].

Interviewer: So how much do you think it would be, do you think?

Irene: Sixteen.

Interviewer: Sixteen? Something like that?

Irene had other knowledge of fifths too. When she was asked to find 20% of the dots in a pattern, she very quickly demonstrated how the dots could be partitioned into five equally sized groups.

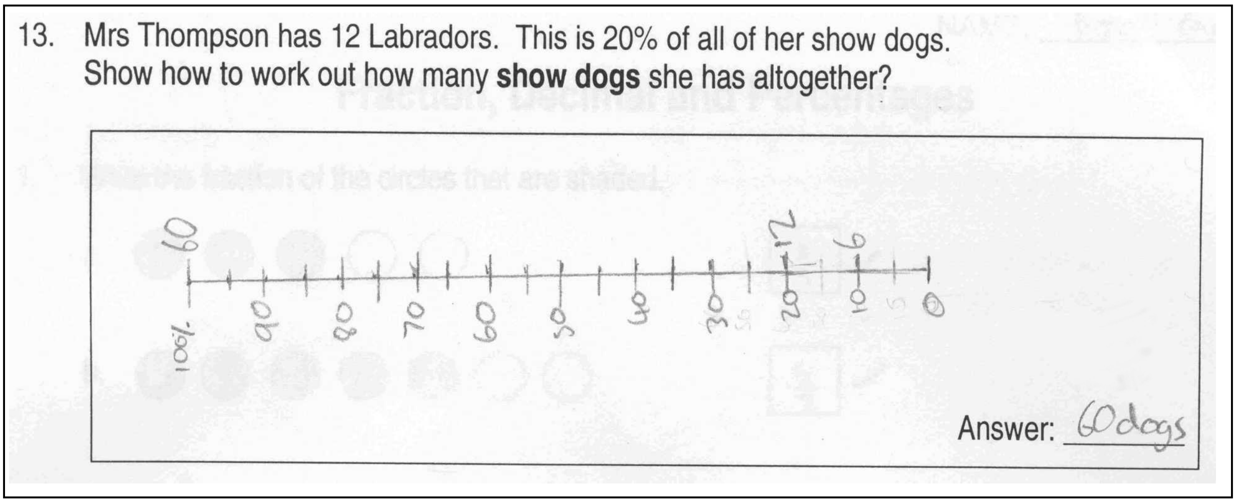
Like Irene, the majority of the students who were interviewed could draw pictures to help represent a problem, or display knowledge that could have been usefully applied to a problem situation. Without prompting from the interviewer however, they were unlikely to do so.

Thinking Models

If students are to become actively involved in their mathematics learning and problem-solving, they must be confronted with learning experiences that encourage them to think and that make their thinking visible to themselves and others. When we were able to present learning experiences that allowed students to develop rich representations of problem situations, we found that the opportunity for students to observe and emulate behaviours that promote SRL was enhanced.

Researchers working in the tradition of Realistic Mathematics Education refer to these kinds of problem representations as models. According to Van Den Heuvel-Panhuizen (2003), models "… reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have different manifestations" (p.13). Models can be based around concrete objects, such as cuisenaire rods, while others can be more abstract, involving systematic ways of organising ideas and data to form a "picture" of a problem situation. Van Den Heuvel-Panhuizen lists materials, visual sketches, paradigmatic situations, schemes, diagrams and symbols as possible models (p.13).

Throughout the teaching experiment, students were introduced to various ways of representing proportional relationships using models. These included double number lines, geometrical shapes, cuisenaire rods, and decimal pipes. One of the most successful models was the double number line. This model allows the elements in a proportional relationship to be modelled graphically. For instance, the double number line below was used by a student to illustrate a problem comparing a part to the whole.



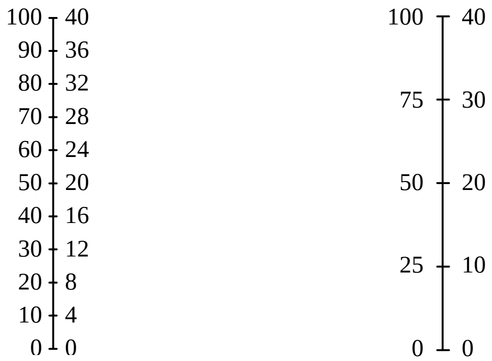
The double number-line was introduced to the students through a series of hands-on activities involving two-litre milk containers. In the first lesson, pairs of students were given a container and asked to construct a scale that could be used to show both the percentage of milk left and the corresponding number of millilitres. Students very quickly applied successive halving to identify the 50 and 25 percent points on the scale and their corresponding millilitre amounts. Some carried on the halving process to mark in the 12.5 percent point. Many of the students also realised that the scale could be turned upside down and the method reapplied to find out where the 75 percent mark would go.

In the next lesson the students were invited to design a scale for a milk bottle company. To satisfy the company's design specifications this scale had to be calibrated in 10 percent sections. It also had to be accurate. It soon became obvious to the students that the halving strategy would not be appropriate in this situation. After some discussion, most agreed that the markings could be located by dividing the length of the scale into ten equal intervals. The students then applied their measurement skills to construct the scale and went on to interrogate how accurate they had been by filling their containers with known amounts of water.

In subsequent discussions, many of the students soon realised that it was now possible to name how many millilitres corresponded to other percentage amounts such as 35 percent, by finding the halfway point between adjacent multiples of ten percent. The scales were now being used to demonstrate methods to find different percentage amounts.

In creating the scales the students had become familiar with how to set up and draw a double number line. At this stage new problems were introduced that were not connected to the milk bottles. New scales, which we now referred to as double number-lines, were drawn to solve problems such as 15 percent of 60. The students were happy to move away from the "concreteness" of the milk bottle scale to the more generalised double number-line. From this point on, regular use was made of double number lines to solve problems and demonstrate thinking. Students used double number lines to explain their solution strategies to their peers and they were constantly referred to by the researchers and the classroom teacher when solutions were modelled to students.

It was also at this point that we were able to observe students in rich discussions about their methods and thinking. Double number lines often became the centre of discussions. Alternative solutions were often demonstrated using the same number line, and strategic decisions justified by appealing to the relationships that had been illustrated. In the transcript below, two students have been working with double number-lines to find 75 percent of 40 dollars. They have drawn two different number lines to solve the problem. In an ensuing discussion with the teacher, the double number-lines assist them to explain their methods and make comparisons between them.



Line 1 *Line 2*

Teacher: Can you tell me how you managed to work this out?

Student 1: Well, for this one we divided it into ten equal bits and we wanted 75%...so we got around about half way [points to halfway between 70% and 80% on line 1], which we figured would be exactly in between these two. We took 28 away from 32, which is 4 and then ... seeing as we only wanted half we did 4÷2.

Teacher: You found the difference?

Student 1: Then we went 32-2=30, which is 75%.

Teacher: What's this one setup for?

Student 1: [Pointing to line 2]. Well, what happened was we halved 40 ... which equals 20. And then we halved 20, which is 10, then you plus 20 + 10, which equals 30.

Teacher: Which one [of these two double number lines] do you think was easier - dividing into tens or dividing into halves?

Student 1: I think it was that one [pointing to line 2] and then that one was close behind though.

Teacher: So, if I said to you say what's 60%, which one would be best to use?

Student 1: 60%? Probably that one [Points to line 1]

Teacher: OK, and if I said 25%, which one?

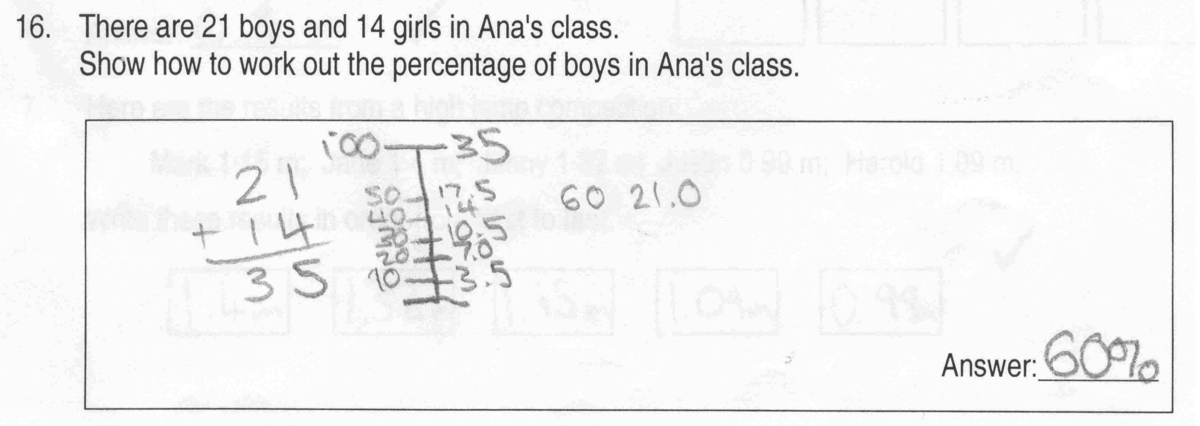
Student 1: That one [points to line 2]

Student 2: That one would be more detailed though [indicates line 1]

Teacher: So, basically whatever percentage you get asked for, can determine what kind of double number line you make …

In this conversation the students are able to clearly articulate their thinking. The number line acts as a kind of scaffold that they can literally point to when explaining their methods. Moreover, the students are able to acknowledge two different types of reasoning that lead to the same answer and even discuss which of the two might be more efficient. The students here are exhibiting metacognitive and reflective behaviours, both of which are essential to SRL.

By the end of the series of twelve lessons many of the students were using the number line spontaneously to attack questions. In the post-test several students employed the number line to successfully attempt problem types that had not been covered in the lessons. Here is one example.



In this example the student has identified the 35 students as representing 100% of the people in the class. She has then worked out that 10% is 3.5 students and built up from there until 21 students (which corresponds to 60%) has been reached. It is interesting to note that the student is happy to label the number line, but is not concerned about drawing the line to scale.

The double number line played an important role in eliciting and revealing thought, thus allowing the students to actively engage in their mathematics learning and problem-solving. In particular it allowed three important SLR skills to be practised.

Firstly, it provided a tool for analysis. The double number line allowed students to analyse the components of the problem and develop a visual representation of the proportional relationships involved. In doing so, it lessened some of the cognitive load involved in problem solving and let them concentrate on observing and controlling the problem-solving process. It also allowed students to explore. Different methods could be recorded or demonstrated on the number lines and the lines used to support reasoning. As students became more familiar with the number lines they recognised strategies that had been applied in analogous problems and attempted to apply them in new situations. Thirdly, students also used the number line to verify their answers. The number-line had to 'look right' if the solution was going to be any good. It also had to convince others.

Models, such as the double number-line invited students to engage in thinking and helped sustain that engagement. As such, they provided opportunities for students to observe and emulate self-regulating behaviours. In the next section, we look at how involving journalling as a complementary part of the classroom routine provided a structured opportunity for students to reflect on their learning.

Journalling

An important phase of SRL involves reflecting on performance to judge progress and make decisions regarding new goals and altered behaviours. It was decided before the lessons began that a process of journalling could provide significant opportunities for students to examine their thinking and reflect on their learning behaviours.

Journalling in mathematics allows students to write about the experiences, ideas and feelings involved in their mathematics learning. At its heart, journalling recognises that writing is a means of "knowing what we think".

> ... writing can engage all students actively in the deliberate structuring of meaning: it allows learners to go at their own pace; and it provides unique feedback, since writers can immediately read the product of their own thinking on paper" (Emig, paraphrased in Borasi and Rose, p.384).

Journalling was used in the classroom on six different occasions. Each time a writing prompt was introduced by one of the authors to stimulate a writing time lasting for approximately ten minutes. The prompts we developed were generally concerned with the students' problem-solving behaviour and conceptual development. For instance, one prompt asked students to write a short explanation for a younger child explaining how to find three fifths of the squares in a five by five grid. In a subsequent session the students read several of their peers' responses and an ensuing discussion looked at what made a good mathematical explanation. Students then commented in their journals on the strengths and weaknesses of their explanations and how they might improve when writing a similar explanation in the future.

Writing itself did not present a barrier for the majority of students in the class. Where it did, the task was altered to suit the child. Students were also encouraged to use diagrams and drawings when needed, to illustrate their writing and were told that the journals would not be assessed for spelling and grammar.

The written feedback by the authors, who read the journals after each session, promoted further reflection. Students were given time to respond to the feedback, which often asked questions about what they had written or requested them to clarify their thinking or provide further examples. Feedback from the journalling was often used to initiate discussion about learning on a whole class or group bases. Moreover, it often helped focus lesson direction and content, often highlighting developing misconceptions and areas of need.

In the example shown a student has written an explanation of how to divide 70 into 10 percent sections. He has illustrated his method with a double-number line. Written feedback from the teacher has resulted in a response in a following lesson. His answer shows he can think

in a flexible manner and is developing increasing sophistication as a proportional reasoner. The journal has provided an opportunity to reflect on his thinking and provided a "window" through which the teacher can observe his increasing range of strategies.

Journalling took time, both to complete in the classroom and for the authors to read the journals before the next lesson. In a busy classroom the practice could be hard to sustain. However, it quickly became evident that the process did not have to occur every lesson. We would argue that the benefits to the mathematics programme and even to writing generally makes this sort of activity very worthwhile.

CONCLUSION

Developing the ability to self-regulate in mathematics learning or as a proportional reasoner does not happen in a vacuum. Supportive classroom environments that nurture the types of thinking and behaviours that support SRL are critical. In this study we have identified models and reflective journaling as two elements of instruction that do just that. Both of these provided students with opportunities to structure and reflect on their thinking and to observe the thinking of others. When they did this, many demonstrated that they could engage in proportional reasoning in an active way.

The work we have done here represents only a tentative start in exploring how SRL might be integrated into mathematics teaching and learning. Overall, we feel that SRL provides a perspective on instruction that is valuable, and that can help promote the kinds of classroom norms, which will support the development of powerful learning and learners.

REFERENCES

Borasi, R., & Rose, B. (1989). Journal Writing and Mathematics Instruction. *Educational Studies in Mathematics, 20, 347-365.*

De Corte, E., Vershaffel, L., & Op' T Eynde, P. (2000). Self-Regulation, A Characteristic and a Goal of Mathematics Education. In M. Boekaerts, P. Pintrich, & M. Zeidner (Eds.), *Handbook of Self-Regulation.* San Diego: Academic Press.

Moss, J., & Case, R. (1999). Developing Children's Understanding of the Rational Numbers: A New Model and an Experimental Curriculum. *Journal for Research in Mathematics Education, 30, 122-147.*

Pape, S., & Smith, C. (2002). Self-Regulating Mathematics Skills. *Theory into Practice, 41* (2), 93-101.

Pape, S., Bell. C., & Yetkin, I. (2003). Developing Mathematical Thinking and Self-regulated Learning: A Teaching Experiment in a Seventh-Grade Mathematics Classroom. *Educational Studies in Mathematics 53, 179-202.*

Pape, S., & Wang, C. (2003). Middle School Children's Strategic Behaviour: Classification and Relation to Academic Achievement and Mathematical Problem Solving. *Instructional Science 31*, 419-449.

Schoenfeld., A. (1992). Learning to Think Mathematically: Problem Solving, Metacognition and Sense Making in Mathematics. In D. A. Grouws (Ed.), *The Handbook of Research on Maths Teaching.* New York: McMillan.

Schunk, D. (1996). Goal and Self-Evaluative Influences During Children's Cognitive Skill Learning. *American Educational Research Journal, 33* (2), 359-382.

Schunk, D. H. (1998). Teaching Elementary Students to Self-Regulate Practice of Mathematical Skills with Modeling. In D. H. Schunk & B. J. Zimmerman (Eds*.), Self-regulated learning: From teaching to self-reflective practice* (pp.1-19). New York: Guilford Press.

Van Den Heuvel-Panhuizen, M. (2003). The Didactical Use of Models in Realistic Mathematics Education: An Example From A Longitudinal Trajectory on Percentage. *Educational Studies in Mathematics, 54*, 9-35.

Zimmerman, B. J. (1998). Developing Self-Fulfilling Cycles of Academic Regulation: An Analysis of Exemplary Instructional Models. In D. H. Schunk & B. J. Zimmerman (Eds*.), Self-regulated learning: From teaching to self-reflective practice* (pp.1-19). New York: Guilford Press.

Zimmerman, B. J. (2001) Theories of Self-Regulated Learning and Academic Achievement: An Overview and Analysis. In B. J. Zimmerman. & D. H. Schunk. *Self-regulated learning and academic achievement: Theoretical perspectives (2nd edition).* New Jersey: Lawrence Erlbaum Associates.

Zimmerman, B. J. (2002). Becoming a Self-Regulated learner: An Overview. Theory into Practice, 41 (2), 64–70.