Te Wāhanga Tuawhā ◊ Chapter Four

Meeting needs and solving equations: Formative assessment and individualised programmes

Robin Averill, Peter Rawlins, Frances Hinchliffe,
Natalie Rutherford and Karen Chow

Lumen accipe et imperti.

Accept the light and hand it on.

Tukua mai, tukua atu.[1]

He kōrero whakataki ◊ Introduction

This chapter describes a variety of ways of using formative assessment to inform teaching. We explore the formative assessment opportunities provided when using activities that develop students' understanding of algebraic expressions and equations.

We begin with a summary of key ideas from the literature about formative assessment. We then share how Frances tailors the school scheme to create individual student learning programmes. Descriptions of flexible learning experiences from Natalie's, Karen's and Frances's classrooms follow, to help show how individual programmes can encompass whole-class and

1 Translation of the Latin. School motto of Wellington Girls' College, Te Kāreti Kōtiro o Te Whanga-Nui-a-Tara.

focused small-group learning, while enabling student autonomy. The examples show how students can be enabled to learn from and with others, putting the school motto of accepting the light and handing it on into practice.

We include the views of some of the students and teachers at the school about the approaches and activities we describe. We finish by revisiting some of the key ideas from the chapter and considering common teaching dilemmas in relation to the implementation of these ideas.

Formative assessment

In recent years traditional views of assessment have been challenged by researchers as they began to understand more about the interactions between assessment and classroom learning. Assessment is no longer seen as an isolated activity or as an endpoint to learning, operating independently of teaching and learning. Instead, assessment is now seen as integral to learning (Wiliam, 2011).

Within the broader assessment literature two main purposes for assessment are discussed. Assessment for summative purposes (assessment *of* learning) is defined as including assessments whose primary purpose is to summarise students' learning at a given point. Assessment for formative purposes (assessment *for* learning) is assessment designed to guide both the teacher and the student in co-developing the next steps in the learning process (Smith, 2010; Wiliam, 2011). Diagnostic assessment, which identifies gaps and patterns of learning strengths and difficulties, is viewed by many as fitting within the area of formative assessment (e.g., Harlen, 2006).

The assessment information gathered from a single task can be used for both summative and formative purposes. Formative assessment includes both formal assessments that take place at pre-specified times and informal assessment that occurs as part of everyday classroom teaching and learning. The main purpose of informal formative assessment is to make students' thinking evident. Teachers can then adapt their teaching to their students' thinking and provide individual feedback and feed-forward (Bishop, Berryman, Tiakiwai, & Richardson, 2003).

Giving students acknowledgement of specific progress made and clear and direct guidance regarding what they need to work on next is known to be a powerful aid for students' learning. Such academically focused one-to-one teacher–student interactions are also important for maximising students' motivation to learn, because they show students that the teacher knows about and cares about their learning.

Informal assessment decisions are often made instantaneously and continuously as teachers make sense of the responses students give. Such decisions are important because they enable the teaching to move in different directions to build on students' existing understanding and meet their learning needs. In mathematics classrooms, informal formative assessment opportunities include using starter questions at the beginning of units or lessons, monitoring students' work, having conversations with individuals or groups, using mini-whiteboards to see students' mathematical thinking, questioning, and using self- and peer-assessment activities. Sometimes these strategies are thought of as "just good teaching" and as "guiding students' learning effectively" rather than specifically as assessment opportunities (e.g., Crooks, 2006). However, research suggests that teachers can be uncertain about how to use formative assessment strategies such as these effectively, with information gathered not always being well used (Carless, 2007; Ruiz-Primo, 2011).

Student engagement in formative assessment is critical for effective learning (Sadler, 1989; Wiliam, 2011). Helping students engage with formative assessment requires teachers going beyond measuring and reporting progress against assessment criteria. If we want students to actively participate, take increased responsibility for and become more autonomous in their learning, we need to prepare them to make the most of formative assessment opportunities (Absolum, Flockton, Hattie, Hipkins, & Reid, 2009; Ministry of Education, 2011). For example, through self- and peer-assessment, students can internalise assessment criteria as they apply them to their own and others' work. Self- and peer-assessment encourage higher-level thinking skills such as metacognition. As students monitor their current understanding, they can self-regulate, plan what to do next, check the outcomes of strategies employed, evaluate, and revise their plan.

Each of the strategies and activities presented next helps to illustrate how students can be engaged in formative assessment as they learn about algebraic equations and expressions. The learning experiences enable students to share responsibility for their learning. We also discuss opportunities the activities present for teachers to gather and use formative assessment information about students' learning to inform their teaching.

Running an individualised programme with deliberate formative assessment opportunities

The school scheme for Number and Algebra lists key content areas, textbook references, activities, web-links and ideas. Before the start of the topic, Frances gives her class a formative assessment task which checks their understanding and skills across the topic. She makes a version of the scheme for each of curriculum Levels 4, 5 and above for students, which includes some of the original scheme information, and key content, websites, problems and puzzles chosen to suit the class (Figure 4.1). Students can access the modified scheme in hard copy or electronically.

	Content	Read, take notes	Exercise Alpha Text	Enrichment
4	**Add/sub fractions (same denominator)**	Read page 99	Ex 7.06 pg 99	Puzzle D15 Puzzle D17
5	**Decimal place values**		Ex 2.01 Pg 16	Puzzle E10 Puzzle 49
6	**Ordering Decimals**		Ex 2.02 pg 19	Decimal Numbers for Ordering P http://www.softschools.com/math/ordering_numbers/ordering_decimals/
7	**Rounding to whole numbers**	Worksheet from folder	Worksheet 1,2 or 3	Decimal Estimates D Calculator colouring 46

Figure 4.1: Excerpt from a modified school scheme.

Each student is also given an individualised 'tick strip' for each level (e.g., Figure 4.2), which identifies which skills they have shown they have mastered (indicated by ticks) and which they need to work on, so that the learning they need to complete is clear. These tick strips act as a useful formative feedback mechanism, clearly indicating the next steps in learning. Having electronic access to the schemes enables students to open embedded web-links and to continue to work through the identified content in and out of class. In most lessons, Frances meets with four or five students to discuss their progress, keeping notes on her own copies of their programmes. Using a different colour each week in her notes enables Frances and her students to visually track progress over time.

Level 4	1. Drawing fractions	2. Fractions from words	3. Equivalent fractions	4. Add/sub fractions	5. Decimal place value	6. Ordering Decimals	7. Rounding	8. Percentage to fraction	9. Fraction to percentage	10. Decimal to percentage	11. Percentage to Decimal	12. Word problems	13. Decimals and money	14. Payments in cash
Stacey	√	√	√			√	√							

Figure 4.2: Stacey's tick strip for Level 4

Frances likes varying her approach, so she would not want to use the individualised strategy for every topic. She has found that for Number and Algebra, the individualised programme works very well for motivating students to take ownership of their learning. Some students ask for further work over and above that indicated on the modified scheme and their tick sheets.

Frances finds that whole-class activities are a valuable part of her individualised programme. They enable her to check that their learning as indicated by the information from assessments—a sample of their understanding and skills—is secure. They help nurture a sense of classroom community and enable students to help one another to understand the topic content and feel confident with it. Frances uses the whole-class activities to observe where students are having difficulty and to have mathematical conversations with individuals, which inform her focus group teaching decisions.

Many of Frances's students respond well to her individualised programme, but others have some reservations:

> Using individual programmes helps me learn as I like going at my own pace and working on what I need to do. The tick sheets help me because I can keep track of what I have been doing and what my progress is. (Hanna)

> I really enjoy this style of teaching. We get to work at our own pace and work on the things we know we need to work on. You don't get bored if you are really smart. (Georgina)

> This was a non-judgemental way of teaching. It had its benefits but sometimes it was hard to understand the book's interpretation. (Stella)

> The individual program does not help me learn because I don't have the motivation to teach myself. Overall I prefer the teacher teaching us and giving us work to do. (Abby)

The students' comments indicate that teachers need to know their students well and put strategies in place to maximise all students' learning. Being aware of the need to energise and encourage the least motivated students and to assist some students with interpreting textbook explanations is important.

Mā hea mai i tēnā ◊ Points to ponder

- Which areas of the mathematics and statistics curriculum are best suited to an individualised approach? Which are least well suited? Why?
- What are the advantages and disadvantages of using an individualised approach?
- Frances used assessment, a modified school scheme and tick strips to set up her individualised approach. What other methods could be used?

Whole-class and focus-group activities

Despite Frances's algebra programme being individualised, her students are encouraged to work together. For example, a lesson might start with a whole-class activity, such as Natalie's 'Loopy on the Go' (below). A small focus group might then work with the teacher using mini-whiteboards, while others choose who to work with on their self- or teacher-selected programme components (e.g., writing algebraic expressions from word problems).

He ngohe ◊ Activity

Natalie's 'Loopy on the Go'

In this activity, cards like this one are displayed around the room. (See Appendix 4.1 for a full set of instructions and cards.)

Each card includes a question and an answer to a different question. Each is labelled with a letter. Students start at any card and find the answer to the question on it, in this case, $4y$. They then locate the card that has the answer they are looking for, and attempt the question on the new card. By finding the answers to the questions, students move around the cards, recording the letters of the cards they have visited in the order they visited them. Checking the order when finished enables them to self-assess. In addition, information at the base of each card shows students where they can find more questions like the one on that card so that they can target their practice on the areas they had most difficulty with.

Natalie likes making her own cards so that she can tailor them to her students' needs. She moves around the room while students are working to gauge their understanding and provide assistance. In addition to the opportunities the activity gives for self-assessment and peers helping peers, she uses this style of activity because it encourages discussion. Students can approach questions individually, in pairs, or in a larger group and there is no beginning or end, so there is less pressure on students to 'complete the task'.

Natalie's experience is that her students find the activity engaging, which indicates that her tailoring of the cards to the students' needs has been successful. Her students find the activities motivating for a range of reasons, including that they are able to make their own decisions about how to work, who to work with, and because it provides useful feedback on their learning:

> It's more interesting than just doing worksheets, because you get to move around and get a variety of questions rather than the same sort of thing repeatedly. You can do it by yourself or with others, and it's an easy way to make sure you're fine on doing most of the things for the test. (Marama)

'Loopy on the Go' is just one type of starter activity. There are many others. Some alternative starter activities include stimulating mathematical argumentation using true/false statements (e.g., "Is this sentence true or false?: $5(x - 7) = 5x - 7$. Explain your reasoning") or creating and describing number patterns (e.g., "What comes next in the pattern 1, 4, 9, 16, 25…?"). Students' mathematical confidence can be enhanced when teachers are strategic in using starter activities to help students avoid common errors. Review questions can be chosen to reactivate prior learning of previous work that is needed for the topic at hand. For example, it is a good idea to review using operations with integers to help make sure students' integer work is secure before working on solving equations.

Using mini-whiteboards

Karen uses a combination of methods to start her lessons. A favourite is using mini-whiteboards to see her students' responses to PowerPoint review questions. Karen finds using the mini-whiteboards good for making students' thinking explicit. They enable her to identify who she will encourage to attend a focus group activity for further development and who to encourage to do other work. They also enable Karen to highlight and acknowledge clear and effective mathematical working. Responses that differ can be used to generate useful mathematical discussion. For example, students can be asked which of a collection of mini-whiteboard answers are true and how they know this. Alternatively, they could be asked to decide and justify which answer shows the most efficient method. Specific mini-whiteboard answers can be chosen for discussion, to help students identify and understand how common errors and misconceptions can be generated and avoided.

He ngohe ◊ Activity

'Let me see your thinking'—mini-whiteboards

Questions and mini-whiteboards can be used to check students' understanding of previous work. For example, Karen's word-based questions (see below) can be used for formative assessment of students' ability to record expressions mathematically.

- Let t represent the number I am thinking of. I double this number and add 8. How would I write this in terms of t?

More of Karen's questions are found in Appendix 4.2.

Karen displays the questions one at a time using PowerPoint so that she can focus on her students and their work. Students display their answers on their mini-whiteboards, one question at a time. As they attempt the questions, students can self-identify for a subsequent focus group, or can be identified by the teacher then encouraged to attend.

Frances also uses mini-whiteboards, both with the whole class and with her small focus groups. Linked to her chosen content focus, Frances will give students four or five questions. For example, for solving equations the questions could be:

$3x - 7 = 20$ $15 - 4x = 3$ $3(x + 7) = 39$ $14x + 12 = 4x - 13$

She then uses selected mini-whiteboard responses as teaching tools to highlight mathematical thinking, elegant solutions or common errors, or to set up discussions about answers that vary.

Figure 4.3: Students with their mini-whiteboards.

The teachers use the whiteboards as efficient formative assessment tools and useful learning tools in all their classes, from Years 9 to 13. The students appreciate using the mini-whiteboards because they know the teacher has seen their individual answers and can respond immediately or later. They like their efficiency in terms of the teacher seeing their mathematical thinking immediately, and they like being able to remove their mistakes. Their comments indicate that they also value the mini-whiteboards as formative assessment tools:

> I love using the whiteboards. It helps me learn because I am tested on hard questions. We use them in small groups, so she can help you improve. (Abby)

> I really like using whiteboards, it is a good way for me to practise a skill so I can check with others, and can then write notes from that. (Fuapepe)

> It's quick, there is more space to figure things out and have the teacher look at what we are doing step by step. (Wei)

> It's an easy way to do the working out without messing it up in your book; if you make a mistake, it's easier to fix it on a whiteboard than in your book. (Aroha)

In addition to being useful for formative assessment, using the mini-whiteboards can help generate whanaungatanga (Bishop et al., 2003) through everyone taking part in the activity together and by enabling students to share responsibility for one another's learning.

Mā hea mai i tēnā ◊ Points to ponder

- Which mathematics and statistics ideas are well suited to teaching and learning that uses mini-whiteboards? Which are not? Why?
- In what other ways can teachers efficiently observe information that is useful for making formative assessment decisions?

Flexible activities within individual programmes

While the focus group is working with the teacher, students not in the focus group can move on to other learning that is included within their individualised programme or that they have identified for themselves as important through the starter activities. Students can ask their peers for help and can find support material in textbooks, on work sheets and through using Internet sites, such as those given as web-links on their modified schemes. Many web-based activities are useful for formative assessment because they enable experimentation and incorporate self-assessment.

One source of such activities is the Illuminations site[2] of the National Council of Teachers of Mathematics. The next activity, found within the Illuminations site, is designed to help develop students' understanding of solving equations.

He ngohe ◊ Activity

'Pan Balance—Expressions'[3]—web-based activities

An example of a suitable web-based activity for developing understanding of numeric or algebraic equations and expressions is 'Pan Balance—Expressions', which can be found in the Illuminations website of the National Council of Teachers of Mathematics.

This interactive activity enables students to practise finding equivalent mathematical expressions for self-generated or peer-generated examples. They can practise algebraic skills and explore the concept of equivalence. The instructions are student friendly.

The Pan Balance activity can also be used by students to move on to exploring graphical forms of linear, quadratic and cubic expressions.

2 http://illuminations.nctm.org/
3 http://illuminations.nctm.org/Activity.aspx?id=3529

Computer sites such as Illuminations enable students to take responsibility for their learning because they can often choose the content or skill focus of the activity and the level of challenge. They can work with others or on their own, and if they have access to a computer outside the classroom they can choose when to work on the activities and how much time they will spend on them. The student decision making enabled through such activities within individualised programmes can help ensure that learning is tailored to students' needs and their personal contexts. When the teacher is not working with the focus group, she or he can monitor students' progress and understanding by roving and questioning.

Whakarāpopoto ◊ Summary

In this chapter we have given examples of how formative assessment activities and tools can be integrated into mathematics teaching and learning. We have seen that specific teacher- and student-generated feedback and feed-forward can be enabled when using activities that lend themselves to gathering formative assessment information. Frances's individualised unit works because she knows her students' individual learning strengths and needs, and shares these with her students in ways that enable them to share control of their learning progress. The individualised approach suits the content area being taught, and students can self-monitor their developing understanding. The lesson activities in students' programmes are varied and include flexible whole-class, focused learning group, collaborative and individual work. Student autonomy is maximised through students knowing what they need to achieve in the topic and how they can access materials. They can decide who to work with and choose which focus group teaching they attend.

Activities that require student choice and mathematical decision making are included in the modified school scheme programmes. Natalie's 'Loopy on the Go' and Karen's mini-whiteboard activities encourage student autonomy and enable self- and peer assessment. In addition, and importantly, students like the individualised approach, the activities and the mini-whiteboards, and how they are used.

Developing understanding of working with algebraic equations and expressions is challenging for many students. Through activities and approaches such as the ones in this chapter, students can work with peers, helping one another to make progress through the unit, thereby reflecting the concept of ako and bringing the school motto "Lumen accipe et imperti" to life.

Ngā pātai kōrero ◊ Discussion questions:
dilemmas of practice

- How can we balance the positive effects of students being able to work with others against enabling the focus group with the teacher to hear and communicate easily?
- How can we maximise the learning of non-motivated students and those not making good progress when using this teaching approach?
- What are the implications for classroom management and monitoring individuals' progress of using the individualised teaching approach described in this chapter?

Ngā rauemi whai ake ◊ Follow-up resources

Other favourite websites for good teaching resources and ideas for number and algebra include:
http://www.mathplayground.com/
http://numberloving.com/
http://www.tes.co.uk/
http://www.mrbartonmaths.com/
For further information about algebraic pedagogical content knowledge, see:

Lawrence, A. (2009). Algebra word problems: A challenge for students and teachers. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice,* Vol. 1 (pp. 113–130). Wellington: NZCER Press.

and

Linsell, C. (2009). Algebra: Students' misconceptions and strategies for solving equations. In R. Averill & R. Harvey (Eds.), *Teaching secondary school mathematics and statistics: Evidence-based practice*, Vol. 1 (pp. 101–111). Wellington: NZCER Press.

Further information about formative assessment is available through the Te Kete Ipurangi web-based assessment community. This site is a useful repository for assessment videos and articles: http://assessment.tki.org.nz/

Ngā huanga tautoko ◊ References

Absolum, M., Flockton, L., Hattie, J., Hipkins, R., & Reid, I. (2009, March). *Directions for assessment in New Zealand: Developing students' assessment capabilities*. Paper presented at the International Symposium on Assessment for Learning, Queenstown.

Bishop, R., Berryman, M., Tiakiwai, S., & Richardson, C. (2003). *Te Kotahitanga: The experiences of Year 9 and 10 Māori students in mainstream classrooms.* Hamilton: Māori Education Research Institute (MERI), School of Education, University of Waikato.

Carless, D. (2007). Conceptualizing pre-emptive formative assessment. *Assessment in Education: Principles, Policy & Practice*, *14*(2), 171–184.

Crooks, T. (2006, June). *Assessment for learning in New Zealand: What's happening and what's next?* Paper presented at the What's Next with Assessment for Learning conference, Massey University, Palmerston North.

Harlen, W. (2006). The role of assessment in developing motivation for learning. In J. Gardner (Ed.), *Assessment and learning* (pp. 61–80). London: Sage Publications.

Ministry of Education. (2011). *Position paper: Assessment*. Wellington: Learning Media.

Rees-Hughes, L., & Derbyshire, S. (n.d.). *Number loving.* Retrieved from http://numberloving.co.uk/

Ruiz-Primo, M. A. (2011). Informal formative assessment: The role of instructional dialogues in assessing students' learning. *Studies in Educational Evaluation*, *37*(1), 15–24.

Sadler, R. (1989). Formative assessment and the design of instructional systems. *Instructional Sciences*, *18*, 119–144.

Smith, K. (2010). Assessment: Complex concept and complex practice. *Assessment Matters*, *2*, 6–19.

Ussher, B., & Earl, K. (2010). 'Summative' and 'formative': Confused by the assessment terms? *New Zealand Journal of Teachers' Work*, *7*(1), 53–63.

Wiliam, D. (2011). What is assessment for learning? *Studies in Educational Evaluation*, *37*(1), 3–14.

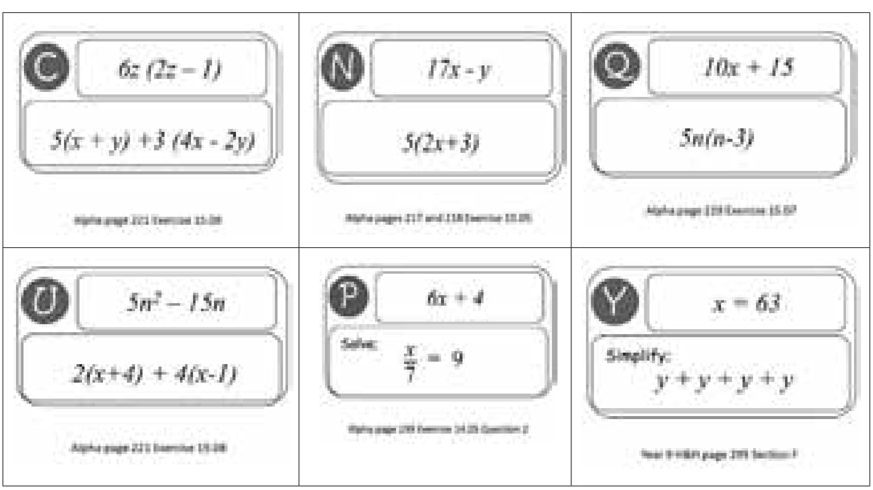
Ngā pūkōrero ◊ Author information

Robin teaches at Victoria University of Wellington, Te Whare Wānanga o Te Ūpoko o te Ika a Māui and Peter at Massey University, Te Kunenga ki Pūrehuroa. Frances, Natalie and Karen teach at Wellington Girls' College, Te Kāreti Kōtiro o Te Whanga-Nui-a-Tara. All are passionate about students being able to share decision making about their learning and using data to inform curriculum decision-making. We are very grateful to students of Wellington Girls' College for sharing their work and ideas with us.

Appendix 4.1: Natalie's 'Loopy on the Go'

Natalie's notes:

- This is a versatile activity that can be used for many topics as a whole class activity or for small groups.
- Keep a copy of the correct letter sequence. In this case it is: C, N, Q, U, P, Y, R, D, G, T, A, V, M, S, W, J, F, H, Z, B, E, I, X, L.
- If using PowerPoint to make the cards, remember to print four or six to a page.
- It is easy to indicate differentiation using colour (e.g., for Achieved, Merit, Excellence; or green set for extension, orange set for target, etc.).
- The original idea came from Treasure Hunt activities at http://numberloving.co.uk/ (see Rees-Hughes & Derbyshire, n.d.), which also has other useful resources.



R $4y$ Simplify: $xy + 2xy + 3xy$ [illegible]	**D** $6xy$ Simplify: $4y^6 \div 2y^2$ [illegible]	**G** $2y^4$ Simplify: $y^3 \times y^5$ [illegible]
T y^8 Simplify: $4x + 7y + 2x - 3y$ [illegible]	**A** $6x + 4y$ Simplify: $5x^2 + 2x - 3x^2 - x$ [illegible]	**V** $2x^2 + x$ Simplify: $4m - 5 + 3m - 6$ [illegible]
M $7m - 11$ Simplify: $2m \times n \times 6$ [illegible]	**S** $12mn$ Solve: $5x = 30$ [illegible]	**W** $x = 6$ Solve: $y + 3 = 10$ [illegible]
J $y = 7$ [illegible] [illegible]	**F** $n = 4$ Solve: $4x - 3 = 9$ [illegible]	**H** $x = 3$ Solve: $x + 2x = 15$ [illegible]
Z $x = 5$ Solve: $4x - 1 = 7$ [illegible]	**B** $x = 2$ Solve: $2x + 3 = 8$ [illegible]	**E** $x = 2.5$ Solve: $7p + 2 = 3p + 8$ [illegible]
I $p = 1.5$ Solve: $5(2y + 3) = 20$ [illegible]	**X** $y = 0.5$ [illegible] [illegible]	**L** $v = 58$ $12z^2 - 6z$ [illegible]

Appendix 4.2: Karen's questions for writing expressions using mini-whiteboards

Let y represent the number I am thinking of. I divide this number by 12. How would I write this in terms of y?	Let y represent the number I am thinking of. I multiply this number by 8. How would I write this in terms of y?
Let t represent the number I am thinking of. I double this number and add 8. How would I write this in terms of t?	Let r represent the number I am thinking of. I subtract 17 from this number. My result is 7. Write an equation for what I have just told you.
Let j represent the number I am thinking of. I multiply this number by 5 and subtract 2. How would I write this in terms of j?	Let s represent the number I am thinking of. I add 5 to this number. My result is 33. Write an equation for what I have just told you.
I am thinking of a number. I divide this number by 8 to give me 19. Write an equation for what I have just told you.	I am thinking of a number. I subtract 8 from this number and halve the result to give me 19. Write an equation for what I have just told you.
I am thinking of a number. When I subtract 4 from this number and multiply this by 5 my answer is 35. Write an equation for what I have just told you.	Jane earns $3 more than Mary. Write Jane's pay in terms of how much Mary gets.
Dina uses three times as much sunscreen as Cleo. Write the amount of sunscreen Dina uses in terms of how much sunscreen Cleo uses.	Hera's Lego collection is a third of Tina's Lego collection Write how much Lego is in Hera's collection in terms of Tina's Lego collection.
Two consecutive numbers are added together. Write down the two consecutive numbers.	Moira earns $3 less than twice the amount Julia earns. Write how much Moira earns in terms of how much Julia earns