DEVELOPING NUMERACY THROUGH Cognitively Guided Instruction

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Worldwide, there is a focus on numeracy that is creating a climate for unprecedented changes in the form and content of teaching mathematics (Askew, 2001). Reform efforts aim to provide mathematical experiences that are radically different from experiences offered in the more traditional classrooms of the past. "Learning mathematics with understanding is thought to occur best in situations in which children are expected to problem solve, reason and communicate their ideas and thinking to others" (Wood, 2001, p.116).

In New Zealand, professional development associated with the Numeracy Project is already having significant impact on classroom practices and student learning outcomes. With regard to arithmetic computations, one critical feature is the focus on students' mental strategies, rather than teacher demonstrations of formal algorithms. Notably, one of the changes reported by teachers has been the increased role of discussion among students of solution strategies for problems within the learning process (Higgins, 2001; Thomas and Ward, 2002).

This paper discusses an established reform programme, Cognitively Guided Instruction (CGI), a professional development programme developed by researchers at the University of Wisconsin (see Carpenter et al., 1999). In CGI, the classroom focus is on establishing a community of inquiry through student discourse that supports emerging understandings of the structural rather than procedural aspects of number. Students are expected not only to present and explain their solution strategies, but also to analyse, compare and contrast the meaningfulness, efficiency and elegance of a variety of strategies. Establishing these classroom norms is not straightforward. Such changes require more than being shown how to implement effective practices. These reforms "require that teachers reinvent their practices so that thinking and learning are interdependent, not separate functions" (Franke et al., 2001, p. 654). They require a willingness on the part of the teacher to listen to the voices of his/her students when attempting to understand their thinking.

CGI is based on the premise that children have a great deal of informal knowledge about mathematics, and that this knowledge can provide a foundation upon which to build children's understanding of arithmetic concepts and skills. The basis for developing children's mathematical understanding and skills is solving a variety of problems. Teachers guide children to use more effective strategies and more complex mathematical representations through a process of problem solving and reflective classroom discourse about mathematical thinking. Students' engagement in justification is central to the learning process: "all students should and can learn from a young age that they do not need to depend on authority or memory to know that what they are learning in mathematics is true and makes sense" (Carpenter, Franke, and Levi, in press).

CGI is not an instructional programme in the sense that it provides teachers with a ready-made script or set of resources. Rather, the programme provides teachers with knowledge, derived from research, about the development of children's mathematical thinking. Teachers learn about the relation between the structure of mathematics and children's thinking of that mathematics. The goal of this approach is that teachers will use their understanding of how children learn mathematical concepts to inform instruction (Carpenter et al., 1999; Franke et al., 2001).

CGI framework

The focus of teachers' knowledge development is a structured framework for understanding the development of students' mathematical thinking. The framework incorporates addition, subtraction, multiplication, and division problem situations. Within this structure, coherent principles relate the semantic structure of problems to the strategies that children use to solve them, and children's strategies evolve in predictable trajectories. For example, distinct types of joining problems (see Figure 1) each represent a different problem to young children.

Each problem varies significantly in difficulty, and children use different strategies to solve each problem. For example, when children begin to solve problems, they will use physical objects such as counters or fingers to directly model the action or relationship described in each problem. For addition problems, direct modelling is distinguished by the child's explicit physical representation of each quantity in a problem, and the action or relationship involving those quantities, before counting the resulting set. Over time, direct modelling strategies are replaced by counting strategies (such as counting on from the larger) and mental strategies using derived facts. Derived fact solutions are based on under-

FIGURE 1. Joining Problem Types

Result Unknown	Tina had 5 marbles. Liam gave her 8 more marbles. How many marbles does Tina have altogether?
Change Unknown	Tina has 5 marbles. How many more marbles does she need to have 13 marbles altogether?
Start Unknown	Tina had some marbles. Liam gave her 5 more marbles. Now she has 13 marbles. How many marbles did Tina have to start with?

standing relations between numbers, and most children will use derived facts before they have learned all the number facts at a recall level. With discussion of alternative strategies, the use of derived facts becomes even more prevalent. These stages of strategies, from direct modelling to using number facts, have parallels in the New Zealand Number Framework (One-to-one counting to Advanced Part-Whole).

Ultimately, CGI promotes a cumulative process of mathematical understanding for the learner. Place value concepts and multidigit operations become natural extensions of the processes children use to solve more basic problems. Operations with multidigit numbers follow a similar pattern of understanding, based on experiences with smaller numbers. Children's development of initial fraction concepts may emerge through the partitioning strategies applied when solving partitive division problems:

> There are 24 children in the class. We want to divide the class into 6 teams with the same number of children in each team. How many children will there be in each team?

The CGI framework emphasises the need to use a range of problem types (e.g., Join, Separate, Part-part-whole, and Compare for addition and subtraction) in order for children to experience an appropriate range of representations necessary for the development of conceptual understanding of number. With multiplication, for example, area and array problems involve a different conception of multiplication to that of repeated addition, grouping/partitioning, rate, and multiplicative comparison problems. A recent New Zealand study (Anthony and Walshaw, 2002) analysed a sample of Year 4 and Year 8 students' understanding of the commutativity property of multiplication (*Does 2 x 5 = 5 x 2? Show me using cubes*). This study noted that by far the majority of students' explanations involved transforming "2 groups of 5" into "5 groups of 2". From the sample of 100 students (randomly selected from the National Education Monitoring Project), only a small number of student responses (7 percent) indicated an approach which suggested mathematical experiencing of arrays. Thus, for young children, who will not normally construct an array representation, it is important that they experience problems in which the array structure is explicitly suggested within the problem context:

> For the meeting, there are 4 rows of chairs with 3 chairs in each row. How many chairs have been put out?

Throughout early number instruction, the fundamental properties that children use in computational problems provide the basis for most of the symbolic manipulation in algebra (Carpenter et al., in press). The focus is not to teach algebraic procedures, but rather to develop ways of thinking about arithmetic that are more consistent with how students have to think to learn algebra successfully. Specific areas of focus within the introduction of arithmetic suggested by Carpenter and colleagues include:

- The meaning of the equal sign (Falkner, Levi, and Carpenter, 1999).
- Development of relational thinking (using true-false and open number sentences).
- Conjectures about number operations, such as commutativity, distributive laws, identity elements, etc.

Within all of these contexts, the goal is not merely to teach students appropriate conceptions of the use of the equal sign or the distributive property, for example; it is equally important to engage them in thinking flexibly about number operations and relations, and in productive mathematical argument. The development of these aspects of students' mathematical thinking is not perceived as one more topic area to teach. Rather, mathematical thinking is an integral part of teaching arithmetic.

A key point here is that teaching is a very situated activity (Loef, Franke and Kazemi, 2001). Teaching which takes as its goal the development of students' mathematical understanding must proceed from an acknowledgement of what students already understand. CGI teachers assign greater importance to engaging in continuous learning about their own students' development of mathematical understanding than to a preordained instructional programme.

What CGI classrooms look like

Even though CGI does not prescribe instruction, CGI classrooms develop common components. Children in these classrooms spend most of their maths time solving problems, and their problem solving strategies are the focus of instruction. Usually, problems are related to a book the teacher has read to the class, a theme or unit being studied outside the mathematics class, or something happening in the lives of the students. Within the context of numeracy, children are encouraged to construct their own efficient algorithms. That is, rather than being shown *how* to solve problems, each child solves them any way that s/he can, often in more than one way.

Solution processes may include using materials, such as manipulatives and/or paper and pencil, or solving a problem mentally. The use of appropriate tools to provide common referents for discussion is an important feature of the CGI numeracy classroom. For younger children in particular, explanations are more likely to be effective in enhancing understanding if students demonstrate and compare their solution methods with the aid of tools (be it cubes, number lines, or symbolic notations), even if they have in fact solved the problems mentally. Such tools enable children to make their underlying cognitive process visible, allowing their solution strategies to become open to public reflection. Later, notations provide a common basis for discussion, helping students to clarify their thinking. The difference between effective and possibly ineffective use of tools is subtle. Instead of using materials to demonstrate the mathematical ideas to be learned, all materials are regarded as tools which a child can select to help solve a particular problem (Carpenter and Lehrer, 1999).

Within the classroom, students must be committed to making sense of their activity and to expressing their sense in meaningful ways. For this to happen, teachers must establish an expectation for students to articulate their thinking. In creating the conditions for personal meanings to become available for classroom discussion, norms for explaining and for listening need to be established. Explanations should include not only descriptions of solutions strategies and/or ideas, but also the thinking and reasoning that led to a solution. In establishing how a problem was solved, students are often asked to justify the solution process and challenged to think more deeply about the underlying mathematical structure. Using questions such as, "How do you know that?" and "Can you prove that?" encourages students to clarify their thinking and to provide justification (Wood, 2001).

In an environment in which each student's thinking is important and respected by the group, the role of the listener (both teacher and student) is especially significant. As listeners, participants need to do more than pay attention and listen politely; they are expected to take an active role and take responsibility for assisting others in making sense of mathematics. Listeners are expected to follow the thinking and reasoning of others to determine whether what is presented is logical and makes sense. Exploring the diversity of opinion, asking clarifying questions, resolving disagreements, and evaluating multiple solutions assists students in their negotiation of mathematical meaning (Gravemeijer et al., 2000).

In this scenario, learning is clearly both an individual and a social activity, with multiple opportunities for rich interactions. As students communicate to their teacher and peers how they solved the problems, the group listens and questions until they understand the strategy that the child is using. The other students share their solution strategies. The entire process is repeated with another problem. Using their own statements, as well as those of their peers and teacher, as thinking devices enables students to acquire a deeper understanding of mathematics.

Listening to students' thinking

Information from each child's reporting of a problem solution assists teachers to make decisions about what each child knows and how further instruction should be structured. They also learn more about possible problems to pose, strategies to expect, and relationships that exist between problems and strategies. However, listening to students' mathematical thinking has another benefit too. The ongoing interactions with students in their own classrooms provide a basis for teachers to create knowledge about children's thinking. Thus, in principle, CGI provides opportunities for teachers to continue learning with understanding. In a New Zealand study involving a CGI model (Anthony, Bicknell, and Savell, 2001), teachers noted that when expectations to share solution strategies were established, it quickly became apparent that young children could often solve arithmetic problems based on their informal knowledge, rather than taught procedures. Significant changes in practice were noted by those teachers who realised that they not only needed to listen to their students' strategies, but also needed to create questions to elicit explanations about their mathematical thinking and justification.

The notion of a framework to support teachers' understanding of children's thinking and developmental trajectory is common to many international numeracy programmes, including the New Zealand Numeracy Project. However, while a framework provides a sound basis for improving teachers' knowledge, research on professional development and teacher change suggests that it does not guarantee sustained and generative change in teaching practices and classroom norms. Research on CGI teacher change suggests that focusing on student thinking is a key component: "Teachers who focus on the principled ideas underlying children's mathematical thinking sustain themselves. Generative growth occurred for teachers who perceived themselves as learners, creating their own understandings about the development of student thinking" (Franke et al., 2001, p. 685).

In summary, the research-based perspective of CGI provides the teacher with a coherent analysis of the structure of the mathematics as well as the developmental strategies that children use when acquiring the ability to solve mathematical problems. The everyday patterns of interaction and the norms that are constituted within a CGI classroom contribute to children's beliefs about the nature of mathematical knowledge and the ways in which one learns and uses mathematics in everyday life. Within this social learning community, each child is perceived to be in charge of his or her own learning, using his or her individual knowledge of mathematics to solve problems that are realistic and relevant. Likewise, teachers are transformed into learners as they engage in practical inquiry within their own classroom and school community.

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